

Periodic Discrete-Time Signals

You said:



What is the condition for discrete time signals to be periodic?

ChatGPT said:



For a discrete-time signal $x[n]$ to be periodic, there must exist a positive integer N such that:

$$x[n] = x[n + N] \quad \text{for all integers } n.$$

The smallest such positive integer N is called the fundamental period of the signal.

In mathematical terms, the condition for periodicity in a discrete-time signal can also be expressed as:

$$x[n] = x[n + kN] \quad \text{for all integers } n \text{ and } k.$$

A key point to note is that N must be an integer, and this distinguishes discrete-time periodic signals from continuous-time periodic signals.

You said:



Sketch the following signal
 $u(t) - u(t-2)$

ChatGPT said:



The signal $u(t) - u(t - 2)$ represents the difference between two unit step functions, $u(t)$ and $u(t - 2)$.

Understanding the Signal:

- $u(t)$: This is the unit step function, which is 0 for $t < 0$ and 1 for $t \geq 0$.
- $u(t - 2)$: This is a shifted unit step function, which is 0 for $t < 2$ and 1 for $t \geq 2$.

Signal Behavior:

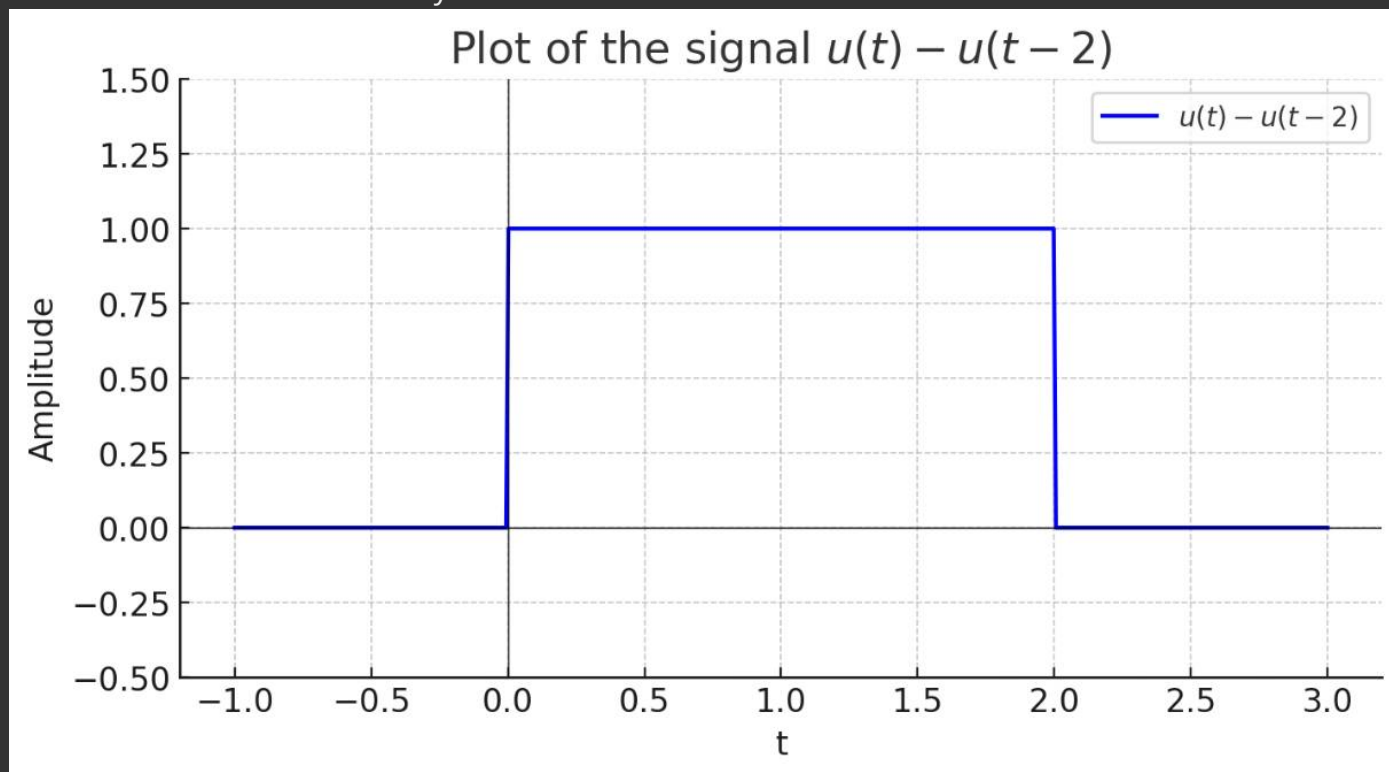
- For $t < 0$: Both $u(t)$ and $u(t - 2)$ are 0, so $u(t) - u(t - 2) = 0 - 0 = 0$.
- For $0 \leq t < 2$: $u(t) = 1$ and $u(t - 2) = 0$, so $u(t) - u(t - 2) = 1 - 0 = 1$.

- For $t \geq 2$: Both $u(t)$ and $u(t - 2)$ are 1, so $u(t) - u(t - 2) = 1 - 1 = 0$.

Resulting Signal:

The signal $u(t) - u(t - 2)$ is a rectangular pulse that starts at $t = 0$ and ends at $t = 2$. It is equal to 1 between these points and 0 elsewhere.

I will now create the sketch for you.



Here is the sketch of the signal $u(t) - u(t - 2)$. As you can see, the signal is a rectangular pulse that starts at $t = 0$, ends at $t = 2$, and has an amplitude of 1 within this interval. Outside this interval, the signal remains at 0.

You said:



A signal $x(t)$ has a fourier transform $X(\omega) = \frac{2}{2 + \omega^2}$, then find $x(t/2)$.

ChatGPT said:



Given the Fourier transform $X(\omega) = \frac{2}{2 + \omega^2}$, we need to find the signal $x\left(\frac{t}{2}\right)$.

Step 1: Identify the inverse Fourier transform

The inverse Fourier transform of $X(\omega)$ gives us the time-domain signal $x(t)$. The Fourier transform $X(\omega)$ matches the form of the Fourier transform of a decaying exponential, which is given by:

$$X(\omega) = \frac{A}{B + \omega^2}$$

The corresponding time-domain signal for this transform is:

$$x(t) = \frac{A}{2\sqrt{B}} e^{-\sqrt{B}|t|}$$

Step 2: Apply the given parameters

For $X(\omega) = \frac{2}{2+\omega^2}$, we have:

$$A = 2, \quad B = 2$$

So, the time-domain signal $x(t)$ is:

$$x(t) = \frac{2}{2\sqrt{2}} e^{-\sqrt{2}|t|} = \frac{1}{\sqrt{2}} e^{-\sqrt{2}|t|}$$

Step 3: Find $x\left(\frac{t}{2}\right)$

Now, we need to find the signal $x\left(\frac{t}{2}\right)$. To do this, substitute t with $\frac{t}{2}$ in the expression for $x(t)$:

$$x\left(\frac{t}{2}\right) = \frac{1}{\sqrt{2}} e^{-\sqrt{2}\left|\frac{t}{2}\right|} = \frac{1}{\sqrt{2}} e^{-\frac{\sqrt{2}}{2}|t|}$$

Final Answer:

The signal $x\left(\frac{t}{2}\right)$ is:

$$x\left(\frac{t}{2}\right) = \frac{1}{\sqrt{2}} e^{-\frac{\sqrt{2}}{2}|t|}$$

You said:



State the time shifting property of Fourier transform

ChatGPT said:



The time-shifting property of the Fourier transform states that if a signal $x(t)$ has a Fourier transform $X(\omega)$, then a time-shifted version of the signal, $x(t - t_0)$, has a Fourier transform that is the original Fourier transform multiplied by a complex exponential factor.

Mathematically:

If $x(t)$ has a Fourier transform $X(\omega)$, then:

$$x(t - t_0) \xrightarrow{\text{FT}} X(\omega) e^{-j\omega t_0}$$

Explanation:

- $x(t - t_0)$: The signal $x(t)$ is shifted in time by t_0 .
- $X(\omega)e^{-j\omega t_0}$: The Fourier transform of the time-shifted signal is $X(\omega)$ multiplied by a phase shift factor $e^{-j\omega t_0}$, where ω is the angular frequency.

This property indicates that a time shift in the time domain corresponds to a phase shift in the frequency domain. The magnitude of the Fourier transform remains the same, but the phase is altered by $-\omega t_0$.

You said:



Determine the complex fourier series coefficients of the signal given by $z(t) = \sin^2 t$

ChatGPT said:



To find the complex Fourier series coefficients of the signal $z(t) = \sin^2(t)$, we'll follow these steps:

Step 1: Express $\sin^2(t)$ using a trigonometric identity

The trigonometric identity for $\sin^2(t)$ is:

$$\sin^2(t) = \frac{1 - \cos(2t)}{2}$$

So, the signal $z(t)$ can be written as:

$$z(t) = \frac{1}{2} - \frac{1}{2} \cos(2t)$$

Step 2: Express $z(t)$ in terms of exponential functions

Using Euler's formula $\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$, we can express the cosine term in exponential form:

$$\cos(2t) = \frac{e^{j2t} + e^{-j2t}}{2}$$

Substituting this into the expression for $z(t)$:

$$z(t) = \frac{1}{2} - \frac{1}{2} \cdot \frac{e^{j2t} + e^{-j2t}}{2}$$

Simplifying:

$$z(t) = \frac{1}{2} - \frac{1}{4}e^{j2t} - \frac{1}{4}e^{-j2t}$$

So, the signal can be rewritten as:

$$z(t) = \frac{1}{2} - \frac{1}{4}e^{j2t} - \frac{1}{4}e^{-j2t}$$

Step 3: Identify the Fourier series coefficients

The complex Fourier series representation is generally given by:

$$z(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

where c_n are the Fourier coefficients and ω_0 is the fundamental frequency. For $z(t)$, the fundamental frequency $\omega_0 = 1$ (since $\sin^2(t)$ is periodic with period $T = \pi$).

From the expression:

$$z(t) = \frac{1}{2}e^{j0t} - \frac{1}{4}e^{j2t} - \frac{1}{4}e^{-j2t}$$

We can identify the Fourier coefficients c_n :

$$c_0 = \frac{1}{2}, \quad c_2 = -\frac{1}{4}, \quad c_{-2} = -\frac{1}{4}$$

For all other n , $c_n = 0$.

Final Answer:

The complex Fourier series coefficients of the signal $z(t) = \sin^2(t)$ are:

$$c_0 = \frac{1}{2}, \quad c_2 = -\frac{1}{4}, \quad c_{-2} = -\frac{1}{4}$$

and $c_n = 0$ for all other values of n .

You said:



Why the Fourier series expansion of the signal $f(t)=\tan(t)$ is not possible?

ChatGPT said:



The Fourier series expansion of a signal $f(t) = \tan(t)$ is not possible because the function $\tan(t)$ is not periodic and has singularities within any finite interval.

Key Points:

1. Periodicity:

- For a Fourier series expansion to exist, the signal must be periodic. The function $\tan(t)$ has a period of π , but it is undefined at $t = \frac{\pi}{2} + n\pi$ for integer n . These points correspond to vertical asymptotes where the function tends to ∞ or $-\infty$. Due to these discontinuities,

$\tan(t)$ cannot be considered a well-behaved periodic function over any interval that includes these points.

2. Discontinuities/Singularities:

- Fourier series requires the function to be integrable over a period. The function $\tan(t)$ has infinite discontinuities (vertical asymptotes) at $t = \frac{\pi}{2} + n\pi$. This causes the function to have infinite values within every period, making it impossible to represent as a Fourier series. The presence of these singularities means the integral of $\tan(t)$ over any interval that includes an asymptote is not finite.

3. Non-Integrability:

- Fourier coefficients are determined by integrating the product of the function and complex exponentials over one period. Due to the vertical asymptotes in $\tan(t)$, these integrals do not converge, and therefore, the Fourier coefficients cannot be defined.

Conclusion:

The function $f(t) = \tan(t)$ does not satisfy the necessary conditions for Fourier series expansion, such as being periodic without singularities and being integrable over a period. Hence, the Fourier series expansion of $\tan(t)$ is not possible.

You said:



A signal $x(t)$ has a fourier transform $X(\omega) = 12/(12 + \omega^2)$, then find the derivative of $x(t)$. i.e. $d/dt x(t)$.

ChatGPT said:



Given that the Fourier transform of $x(t)$ is:

$$X(\omega) = \frac{12}{12 + \omega^2}$$

we are asked to find the Fourier transform of the derivative $\frac{d}{dt}x(t)$.

Step 1: Fourier transform of the derivative

The Fourier transform property for the derivative states:

$$\mathcal{F} \left\{ \frac{d}{dt}x(t) \right\} = j\omega X(\omega)$$

This means that the Fourier transform of $\frac{d}{dt}x(t)$ is $j\omega X(\omega)$.

Step 2: Compute the Fourier transform of $\frac{d}{dt}x(t)$

Using the given $X(\omega)$:

$$\mathcal{F} \left\{ \frac{d}{dt} x(t) \right\} = j\omega \cdot \frac{12}{12 + \omega^2}$$

Step 3: Inverse Fourier transform to find $\frac{d}{dt} x(t)$

Now, we need to determine the inverse Fourier transform of $j\omega \cdot \frac{12}{12 + \omega^2}$. To do this, we recognize that the term $\frac{12}{12 + \omega^2}$ is associated with a time-domain function, and the multiplication by $j\omega$ corresponds to the differentiation in the time domain.

We know that the inverse Fourier transform of:

$$\frac{12}{12 + \omega^2}$$

is:

$$x(t) = e^{-\sqrt{12}|t|}$$

where the coefficient and the time-domain representation match that of a decaying exponential.

Since the Fourier transform of $j\omega X(\omega)$ is the time-domain derivative:

$$\frac{d}{dt} x(t) = \frac{d}{dt} \left(e^{-\sqrt{12}|t|} \right)$$

Step 4: Compute the derivative

Given that $x(t) = e^{-\sqrt{12}|t|}$, the derivative is:

For $t \geq 0$:

$$\frac{d}{dt} x(t) = -\sqrt{12} e^{-\sqrt{12}t}$$

For $t < 0$:

$$\frac{d}{dt} x(t) = \sqrt{12} e^{\sqrt{12}t}$$

Final Answer:

The derivative $\frac{d}{dt} x(t)$ is:

$$\frac{d}{dt} x(t) = \begin{cases} -\sqrt{12} e^{-\sqrt{12}t}, & \text{for } t \geq 0 \\ \sqrt{12} e^{\sqrt{12}t}, & \text{for } t < 0 \end{cases}$$

ChatGPT can make mistakes. Check important info.